MAT 2377
Supplemental Examination
Winter 2007

Time: 3 hours

Student Number:

Family Name: __ First Name: ____

This is an open book examination and the faculty of engineering standard calculator is allowed. Please circle the correct multiple choice answer. There are no booklets so write the answers to part B in the space provided on the examination paper.
Part A: 13 questions (7 marks for each question)
Part B: 22 marks.

Question A-1 If $E(X^2) = E((X-3)^2) = 5$ for some random variable X, find its variance σ_X^2.

Answer: (a) 2.75 (b) 3 (c) 3.25 (d) 3.5 (e) None of these answers.

Question A-2 Let A and B be two independent events such that $P(A) = 0.6$ and $P(B) = 0.2$. Determine $P(A \cup B^{'})$.

Answer: (a) 0.92 (b) 0.52 (c) 0.68 (d) 0.72 (e) None of these answers.

Question A-3 From an announcement by Air Canada it is known that 10% of clients reserve first class seats. Among the next 5 reservations, what is the probability that none of them reserves a first class seat if reservations are made independently ?

(a) 0.00001 (b) 0.9999 (c) 0.5905 (d) 0.4095 (e) None of the previous answers.

Question A-4 Let X be a normally distributed random variable with mean μ and variance $\sigma^2 = 4.84$. Determine the size of the sample required to be 90% confident that the error in the estimation of μ is at most 0.4.

Answer: (a) 163 (b) 117 (c) 82 (d) 11 (e) 10

Question A-5 Earthquakes in a given region follow a Poisson process with rate 2 per week. What is the probability that there will be at least 3 earthquakes in the next 10 days?

Answer: (a) 0.323 (b) 0.679 (c) 0.601 (d) 0.544 (e) None of these answers.

Question A-6 If Y is a random variable with p.d.f.

$$f(y) = \begin{cases} c(1-y), & \text{if } 0 < y < 1 \\ 0 & \text{if Otherwise} \end{cases}$$

Then $Var(Y)$ is

Answer:
(a) 1 (b) 2/3 (c) 1/18 (d) 3/4 (e) 1/5.

Question A-7 In a given survey about consumer preference between 2 brands of a product, 59 out of 112 people interviewed preferred brand A over brand B. Let p be the true proportion of the population that prefer brand A. If we test the hypothesis $H_0 : p = 0.5$ versus $H_1 : p > 0.5$, what is the p-value of our result?

Answer: (a) .142 (b) .284 (c) .5 (d) .526 (e) .567

Question A-8 A certain brand of candy bar is supposed to weight 25 grams. We assume that the bar weights follow a normal distribution with standard deviation of 3 grams. We buy 4 such bars and compute the mean weight in order to test $H_0 : \mu = 25\text{g}$ versus $H_1 : \mu < 25\text{g}$. If we define our critical region to be $C : \overline{x} < 22.5$, what is the probability of a type I error of our test?

Answer: (a) .1 (b) .07 (c) .0475 (d) .0238 (e) None of these answers.

Question A-9 The yield per acre of a certain variety of grain is normally distributed with mean μ and variance σ^2. Given the following 12 measures (in tons):

$$\{31.5, 29.9, 27.7, 31.2, 24.6, 38.5, 29.9, 38.7, 40.6, 29.4, 35.3, 32.0\}$$

what is the length of a 90% confidence interval for the mean μ?

Answer: (a) 2.31 (b) 2.51 (c) 3.82 (d) 4.61 (e) 5.04

Question A-10 In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $p = 0.7$ be the probability that the student knows the answer and $q = 0.3$ the probability that the student guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{5} = 0.2$. What is the conditional probability that a student knew the answer to a question, given that he or she answered it correctly ?

Answer: (a) 1/6 (b) 50/86 (c) 70/76 (d) 20/30 (e) none of the preceding

Question A-11 Consider the random variables X and Y with joint probability mass function:

$$f(x, y) = c(x + y), \ x \in \{1, 2, 3\}, \ y \in \{1, 2\}$$

and 0 otherwise. What is the value of c and are X and Y independent or not?

Answer: (a) $c = 1/9$, independent (b) $c = 1/21$, independent (c) $c = 1/9$, dependent (d) $c = 1/21$, dependent (e) None of these answers.

Question A-12 Let X equal the grade of a student in a statistics course. Suppose that X is $N(\mu, 100)$. Using the traditional method of teaching, the average student grade was $\mu = 60$. We want to test if the the students' grades in this course has increased due to a new teaching method. In order to test $H_0 : \mu = 60$ versus $H_1 : \mu > 60$ we will use a random sample of $n = 36$ students. If $\overline{x} = 63.5$ is the average grade of this sample, find the p-value.
Answer:

(a) 0.0237

(b) 0.0179

(c) 0.0537

(d) 0.0968

(e) None of the previous answers.

Question A-13. A picker is asked to gather 36 ripe apples and put them in a bag. If $X_i, i = 1, 2, \ldots 36$ is the weight of apples and the weight of an individual apple has mean $\mu = 0.2$ pound and the standard deviation of $\sigma = 0.04$ pound, use the central limit theorem to find

$$P(\sum_{i=1}^{36} X_i \leq 8.6)$$

(a) 0.9803 (b) 0.8508 (c) 0.6915 (d) 1 (e) none of the preceding.

Question B-1 The number of soldering defects per device was measured over several months. The empirical histogram of the number of soldering defects per device is approximately given by the following probability mass function $f(x)$:

x defects	0	1	2	3	4	5
$f(x)$	0.1	0.3	0.3	0.1	0.1	0.1

a) Calculate the mean number of defects per device.

b) Calculate the standard deviation of the number of defects per device.

c) Each soldering defect takes about 10 minutes to fix. If we ship the customer 50 devices, what is the probability a repair man will take more than an 8 hour shift to fix all the defects.

Question B-2 Our old plastic moulding machine produces parts with a mean breaking strength of 1000 kilograms with a standard deviation of 100 kilograms. A new model has come out which promises to produce parts with a mean breaking strength of more than 1000 kilograms and a standard deviation of 100 kilograms. Money is tight so the boss will only authorize the purchase if there is a high degree of certainty the claims about the new model are true. We can run a trial of the new machine but each housing is very expensive so we can only test a few. They make five moulds with the new machine and the five breaking strengths are

$$1022, 1230, 1187, 1322, 1166.$$

a) What is our null hypothesis? What is the alternative? If the new model is really no better than the old but we get the boss to buy the new one, what type of error are we making?

b) Perform the test at level .01

c) If the new moulding machine follows the specifications of the manufacturer but we advise the boss to stick with the old machine what kind of error are we making? What is the probability of this error ?